EECS 360 Signal and System Analysis

Lab 7. Approximation of the Continuous Time Fourier Transform

The continuous time Fourier Transform can be approximated by the following sum:

$$\int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt = \lim_{\tau \to 0} \sum_{n=-\infty}^{\infty} x(m\tau)e^{-j2\pi f_n \tau}\tau$$

Given that you have a record of T seconds sampled every τ seconds resulting in a total of N samples (Note: $T = N^* \tau$ of the signal $x(t)$ then the continuous time Fourier Transform can be approximated by:

$$\int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt = \int_{0}^{T} x(t)e^{-j2\pi ft}dt \approx \sum_{m=0}^{N-1} x(m\tau)e^{-j2\pi f_m \tau}\tau$$

Let

$$f_0 = \frac{1}{T} = \frac{1}{N\tau} \quad \text{so} \quad f = nf_0 = \frac{n}{N\tau}$$

and define

$$X_n = \sum_{m=0}^{N-1} x(m\tau)e^{-j2\pi nf_0 m\tau}\tau = \tau \sum_{m=0}^{N-1} x(m\tau)e^{-j2\pi nm/N}$$

Now X_n can be viewed as an approximation for the continuous time Fourier Transform of $x(t)$ at $f=nf_0$. Note that X_n is a complex number.

a). Given $x(t)$, T, τ, and write a Matlab routine to find X_n.
b). Test and validate your routine for $x(t) = u(t)e^{-t}$ with $\tau = 0.01$ sec and $T = 1.28$ sec by analytically determining the Fourier Transform for $x(t) = u(t)e^{-t}$ and graph $|X(f)|$ and $|X_n|$ on the same plot, also graph the phase of $X(f)$ and X_n on the same plot.
c). Repeat part b) for $T = 5.12$ sec.
d). Repeat part b) for $\tau = 0.1$ sec and $T = 12.8$ sec.
e). Comment on the accuracy of the approximation as T increases.